

#### BAULKHAM HILLS HIGH SCHOOL

## - TRIAL 2012 YEAR 12 TASK 4

# **Mathematics**

#### General Instructions

- Reading time 5 minutes
- Working time 120 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11-16
- Marks may be deducted for careless or badly arranged work

Total marks – 100 Exam consists of 7 pages.

This paper consists of TWO sections.

Section 1 - Page 2 (10 marks)
Questions 1-10

Attempt Question 1-10

Section II - Pages 3-6 (90 marks)

• Attempt questions 11-16

Table of Standard Integrals is on page 7

### STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a \neq 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 + a^2}) \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: 
$$\ln x = \log_e x$$
,  $x > 0$ 

#### Section I - 10 marks

Use the multiple choice answer sheet for question 1-10

 $x^3 + 27 =$ (A)  $(x+3)(x^2+6x+9)$ 

(B)  $(x+3)(x^2-6x+9)$ (D)  $(x+3)(x^2-3x+9)$ 

(C)  $(x+3)(x^2+3x+9)$ 

- What is the exact value of  $\cos \frac{7\pi}{2}$ 
  - (A)  $\frac{\sqrt{3}}{2}$

- (D)  $-\frac{1}{2}$
- 3. For the quadratic equation  $y = ax^2 + bx + c$ , if  $\Delta > 0$  and a > 0 then the parabola is (C) positive definite (D) has one root (B) negative definite (A) indefinite
- $e^{2\log_e x}$  is equal to
  - (A)  $\sqrt{x}$
- (B)  $x^2$

- 3x 5 dx =
  - (A) 10
- (B) -10

- (C)20
- (D) -20
- The following table list the values of a function y = f(x) for 3 values of x

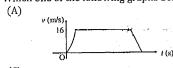
ſ	x	1.2	1.3	1.4
	у	3	3.8	4.2

By using the Simpsons rule and these values, the best estimation of

- (A) 0.75
- (B) 1.12
- (C) 1.49
- (D) 2.24

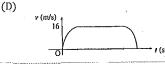
- If  $f(x) = a^{x} + a^{-x}$ , then  $f^{2}(x) =$ 
  - (A) 1 f(2x)
- (B) 1 + f(2x)
- (C) 2 f(2x)
- (D) 2 + f(2x)
- For the geometric series  $a(m+1) + a(m+1)^2 + a(m+1)^3 + \cdots$  to have a limiting sum The condition is:-
  - (A) -1 < m < 1
- (B) -2 < m < 2
- (C) -2 < m < 0
- (D) 0 < m < 2
- 9. A particle, initially at rest at the origin, moves in a straight line with velocity  $V = 6 - 2t \, m/s$ . To the nearest metre, the total distance travelled in the first 6 seconds is (D) 18m (A) 0m (B) 6m (C) 9m
- 10. A van starts from a check point, accelerating initially at  $4 m/s^2$  but with the acceleration decreasing until a maximum speed of 16 m/s is attained. It continues at 16 m/s for some time, then slows with a constant deceleration until it comes to rest.

Which one of the following graphs best represents the motion of the van?





(C) v (nvs)



End of Section 1

Section II - Extended Response

Attempt questions 11-16.

Answer each question on a SEPARATE PAGE. Clearly indicate question number.

Each piece of paper must show your name.

All necessary working should be shown in every question.

Qu	estion 11 (15 marks)	Marks
a)	Express 63° in radians to 3 significant figures	2
b)	Sketch the curve $y = \log_e x$ . State its domain and range.	3
c)	Given $(3 + \sqrt{2})^2 = a + \sqrt{b}$ Find $a$ and $b$ .	2
,	Solve for $x$ $ x-3 =2$	2
e)	Find the vertex and focus of $(x+2)^2 = -12y + 6$	2
f)	A ship sails from a Port A 50 nautical miles due east to a Port B.  It then proceeds a distance of 30 nautical miles on a bearing of 020°T to Port C  (i) Find the distance of Port C from Port A (correct to 2 decimal places)  (ii) Find the true bearing of Port C from Port A	2 2

Qu	estion 12 (15 marks)	
a)	In your number booklet, plot the points $A(-2,1)$ , $B(3,3)$ , $C(2,6)$ and $D(-3,4)$ on a number plane.  (i) Show that the equation of $AB$ is $2x - 5y + 9 = 0$ (ii) Find the exact perpendicular distance from the point $C$ to the line $AB$	2 2
	(iii) Hence find the area of the parallelogram ABCD	2
b)	An arc length 7 units subtends an angle $\theta$ at the centre of a circle of radius 3 units. Find the value of $\theta$ to the nearest degree.	2
c)	Differentiate the following functions with respect to $x$ (i) $\cos 5x$ (ii) $e^x \ln x$	2
d)	The shaded region in the diagram is bounded by the curve $y = x^2 + 2$ , the $x$ -axis, and the lines $x = 0$ and $x = 2$ .	3
	Find the volume of the solid of revolution formed when the shaded region is rotated about the x-axis.	

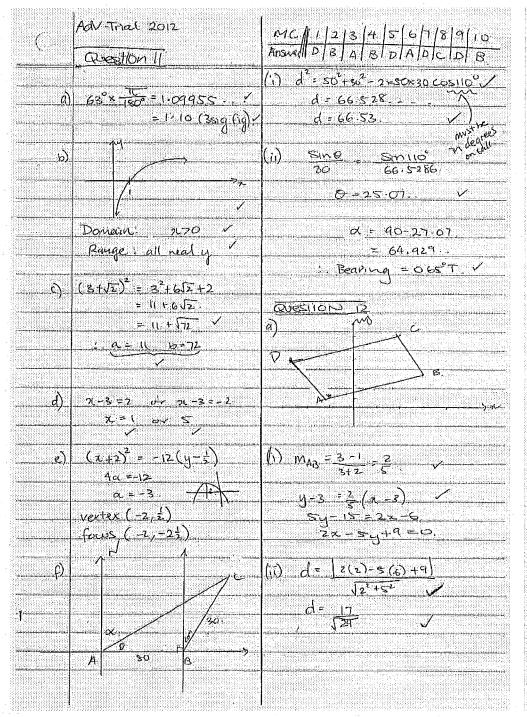
Que	estion 13 (15 marks)	Marks	
a)	Find the equation of the tangent to the curve $y = x^4 - 4$ at the point $(1, -3)$		
b)	Given $BE = 3$ units, $AE = 4$ units and $AB = 5$ units and $CD = 7$ units.		
	Copy the diagram in your booklet:  (i) Prove that $\triangle ABE$ is similar to $\triangle ADC$ .  (ii) Find the exact length of $BC$ .	2 1	
c)	A bottle of solvent is open and the solvent evaporates in such a way that the amount remaining, $V$ $ml$ , in the bottle is given by $V = 1500e^{-0.003t}$ , where $t$ is time in hours.		
	(i) How long is it before half the initial amount of solvent has evaporated from the bottle?		
	(ii) If the solvent continues to evaporate will the bottle ever become empty? Explain.	2	
d)	If $\alpha$ and $\beta$ are the roots of the quadratic equation $\frac{1}{x} = x + p$ ,		
	(i) Show that the quadratic equation can be written as $x^2 + px - 1 = 0$		
	Find in terms of p:-		
	(ii) $\alpha + \beta$	1	
	(iii) $\alpha\beta$ (iv) $2\alpha^2 + 2\beta^2$	1 2	
e)	The acceleration of a moving body is given by $a = \sqrt{2t+1} \ ms^{-2}$ .  If the body starts from rest, find its velocity after 4 seconds.		

Duestion	14	(15 marks	`

Que	SHOIL 14 (15 marks)	
a)	Given that	1
•	$f(x) = \begin{cases} 2 & for \ x < -1 \\ 6x - x^2 & for \ x \ge 0 \end{cases}$	1
	$\int (x) = \begin{cases} 6x - x^2 & for \ x \ge 0 \end{cases}$	
		ĺ
	(i) Find the value of $f(f(-1.1))$	2
	(ii) Find the vertex for the equation $y = 6x - x^2$	1
	(iii) Graph $y = f(x)$ , clearly showing all important features.	3
	(iv) Hence or otherwise, solve for $f(x) = 2$ , leaving your answer in exact form.	2
b)	Solve for $x$ for $0 \le x \le 2\pi$	3
	$2\sin^2 x = 3(\cos x + 1)$	<u> </u>
c)	Explain why the curve $y = \frac{3x+5}{x+1}$ is always decreasing for all real values of x except	2
9 4.	x = -1	-
d)	Simplify	2
-	$\frac{14}{\nabla}$	
	$\sum_{n=1}^{24} (ax+2)^{x-13}$	
	<del>x=13</del>	l

Que	estion 15 (15 marks)	Marks	
a)	It is given that $f(x) = -f(-x)$ , where $f(x)$ is continuous. Find the value of $\int_{-5}^{5} f(x) dx$ and justify your answer.		
b)	The graph $f(x) = a \sin(2x + b) + 3$ is shown in the diagram:  (i) State the period.  (ii) Find the values of $a$ and $b$ .	1 2	
c)	(iii) How many solutions are there for $a \sin(2x + b) + 3 = 0.5x + 1$ For the graph $y = f(x)$ , it is given that $f(-2) = 0$ , $f(0) = 2$ and $f(x) < 3$ Below is a graph of $y = f'(x)$ . Draw a possible graph for $y = f(x)$ showing information known.	2	
d)	A dealership offers Ashwin \$20000 loan for a new car at the beginning of the year for which it charges interest at the rate of 6%pa.  (i) Ashwin wants to make monthly repayments at the end of the month and pay it off in 5 years.  Let M be Ashwin's repayments	The second secon	
-	( $\alpha$ ) Show that the balance of Ashwin's account at the end of the 2 <sup>nd</sup> month is $A_2 = 20000 \times 1.005^2 - 1.005M - M$	.1	
	( $\beta$ ) Find an expression for the amount owed at the end of the 5 years and deduce that $M = \frac{0.005(20000 \times 1.005^{60})}{1.005^{60} - 1} = $387 \text{ (to the nearest dollar)}$	2	
	<ul> <li>(γ) Show that at the end of the first year Ashwin will owe the dealership \$16460 (to the nearest ten dollars)</li> <li>(ii) At the beginning of the 2<sup>nd</sup> year, Ashwin's mother decides to secretly make \$101</li> </ul>	T.	
	repayments at the beginning of the month to help him pay it off earlier.  How many payments did Ashwin's mother have to make until it was paid off?	3	

Qu	estion 16 (15 marks)	Marks
a)	If $a = \log_{\frac{1}{x}} \frac{1}{N}$ such that $x > 0$ and $N > 0$ , show that $a = \log_x N$	
b)	The sum of a series is given as $S_n = n^2 - 3^n$ .	
:	(i) Find $S_{12}$ , leaving your answer in 2 decimal places.	1
	(ii) Find the 12th term of the series in 2 decimal places.	2
c)	(i) Show that $\frac{d}{dx}(\sec x) = \sec x \tan x$	2
	(ii) Show that $\frac{d}{dx}(\sec x + \tan x) = \sec x (\sec x + \tan x)$	1
	(iii) Hence, or otherwise, evaluate	3
	$\int_{\pi}^{\frac{\pi}{3}} \sec x  dx$	and the second
	5	
d)	The triangle $PDB$ with fixed side lengths $b$ and $d$ is given below.	
	$\frac{b}{b}$ $\frac{\pi}{6}$ $\theta$ $\theta$	
	(i) Show that the area, A, of a triangle DBP is given by $A = \frac{1}{2} db \sin \left( \frac{5\pi}{6} - \theta \right)$ .	1
	(ii) Hence prove that the maximum area of the triangle occurs when $b=\sqrt{3}d$	3
		1



		10 <del></del>
(	Question 12 (cont)	Question B
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	Area = 17 , 127	$at x=1 m_{r} = t \times 1$
	=17 mits /	y+3 = 4(x-1) y = 4x - 7
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	0 = 2 rachars.	(b) (5)
	9:139 ::	5
	<u>er</u>	b) B C 4
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	Ø=134° /	DABE is right angled trangle
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	) of coss = = 5 sin520;	1: <u>2</u> NCF (0
101-210000000		In AABE \$ AADC
uranton eagle, t	11) $u = e^{x}$ $v = \ln x$ ? $u = e^{x}$ $v' = \frac{1}{2}$	LBEA = CACD = 90°
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	$\frac{d e^{x} \ln x = e^{x} \ln x + \frac{e^{x}}{x}}{dn}$	
	ch -/ -	ii) AC CP (matching sides  HE BE (af Smilar D's in rat
, (lo	V=π (x+2) dn v	BLIS- 7
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	$V = \pi \int_{0}^{\frac{\pi}{2}} x^{4} + 4\pi^{2} + 4 d\pi $	+ BC = 13 , 3., /
	= 10 2 + 42 + 42 /	·
erener kanaa	and the control of th	
nu mu (55	$= \frac{1}{5} \left( \frac{2^{3} + 3 \times 3}{5} + 4 \times 1 - (0) \right)$ $= \frac{376 \times 1}{15} = 178.75$	
	= 376it = 78.75 V	

Question 13 (cont) V = 1500 e-0003t e) v= (26+1) dt. When t=0, V=1500 V= 2 (24+1)= 3k2 750=1500e-010034 at t=0 v=0loge (05) = -003+ c = - ± t= 2311049 hrs. V== 1 (26+1) == 1 (30(4) / .lm 1500e-0000st V=9-3  $V = \frac{26}{2} \text{ ms}^{-1}$ \$0 but never = 0 Question 14 Since v=0 is the asymptote (i)  $f(-i\cdot i) = 2$ . if will never be empty f (f(1.D) = f(2)  $= 6(2) - (2)^{2}$  = 8(ii) axis of sym  $\Rightarrow x = \frac{-6}{2x-1}$ .  $1 = x^2 + px,$   $x^2 + p(-1) = 0$ when x = 3y = -3 2 1-6x3 ii) 213 = -b = -p-, : Vertex 13 (3,9) 10) QB=-1. (11) Yentex / "" Harter Ex 2 7  $|V| = 2 \left( |x^2 + \beta^2 + 2 \times \beta + 2 \times \beta \right),$   $= 2 \left( |x + \beta|^2 - 2 \times \beta \right)$   $= 2 \left( (-\beta)^2 - 2(-1) \right) /$   $= 2 \rho^2 + 4 \rho^2 /$ (-6-6) 62-107=2 22-6x+2=0 26= 61528 16=8=11 ON XX-1 /

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B	2((-cos2x)=3cox+3.	b)(i) perco(= 2 = 1c :/
	2652x+365x+1=0	
	(2605x+1) =0	$\frac{ok}{(7)}a=2$ . $\frac{a=-2}{a=-2}$ .
	COST = - 1 or USX = -1	2=25in(2(0)+b)+3, 2=25in b+3
·	Auster-3	-1=5nb, t1=5inb
t titl til en kenten ster sid sid.		$\begin{vmatrix} -\frac{1}{2} = Sinb, & \frac{+1}{2} = Sinb, \\ b = \frac{\pi}{2} & b = \frac{\pi}{2} \end{vmatrix}$
	2 21 41 or IL	6 2
		(ii) Seolns.
c)	U=3215 V=2+1	0 / 7 3
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-1	dy 3(x+1)- (3x+5) dn (x+1) <sup>2</sup>	(-5)
	2 - 2 (2.ft) <sup>2</sup>	2 )n
· · · · · · · · · · · · · · · · · · ·	(n+1) <sup>2</sup>	
	Since cotto 20	a) P=20000' P=0.06pg n=5yps  P=0.005 primates n=60mg  (x) A_1 = 20000 × 1.005 TM
**************************************	and -2 10	R=0∞spmath n=60mo
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) distribution = 1 - 1 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 -	<b>.</b>	
a()	$\sum_{N=13}^{14} \frac{(\alpha N+2)}{(\alpha N+2)}$	Az = 20000×1-0052-1-00514-M
		3342332423
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		0=20000 X1005 6-11/(1005 4+1)
	Question 15	20000x (.005 0= M (.1(1.005 0=1))
		20000x (.005 60 = M ( 1 (1:005 60-1) )
a)_	$\int_{\mathbb{R}}^{\infty} f(\omega) = 0$	[ (-005 -1 ] V
······································	——————————————————————————————————————	0.005 (2000×1.0050) = M(1.0050)
	Since f(n) is an odd /	
	function, the orea above	1:002 (30000 × 1.002 (60)
dammania dali basil	cancels with area below.	/-005 <sup>66</sup> -/
		£\$3&7 <u>.</u>

allestion 15		( auct)
$Y) A_{12} = 200000 \times 1005^{12}$	ii) of (seca Hanx)	16d(i) d2A = 1 bol sin (51 0)
-387 (1.05°2-1) A12 -0616459, 693 -	. = Secontaun + sec <sup>2</sup> n	whom $\sigma = \frac{1}{3}$
A12 =4 16460 V	= Secultan il + seczi)	$\frac{d^{2}A}{d\theta^{2}} = \frac{1}{2} bd \sin \left(\frac{\pi}{2}\right)$
(ii) see last <u>rage</u>	-ii) J <sup>17</sup> / <sub>3</sub> sec x x tanx+secx on tanx+secc	= ½bd Since bd > 0 then
alestron 16	(A)	$\frac{d^2A}{d\theta^2} < 0  \nearrow$
i) a = Log, #	= [In(t-anguitseon]] \[ \frac{1}{3} \]	20 max at 0 = 17-
(元) = 二	$\Rightarrow \ln\left(\tan\frac{\pi}{3} + \sec\frac{\pi}{3}\right) - \ln\left(\tan\frac{\pi}{6} + \sec\frac{\pi}{6}\right)$	
$\frac{1}{X^n} = N$	= In (15 F1;) - In (15 + 25)	: b d 임n(등) 임n(등) ✓
	= (n (53t1)-ln ( * 5)	2x(3) (2) x 2
$x^{a} = N$ $= a = \log_{10} N /$	= ln ((3+1)-ln (3=15) = ln ((3+1)-ln (J3)	b = d V8 1
(i) $S_R = 12^2 - 3^{12}$ = $-5^2 31247$	= [4 (\frac{\sum_2 +1}{\sum_2})	b=d√s.
= -5-31247 /	= 6:767651	
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$(i)$ $T_{n_2} = S_{n_2} - S_{n_3}$	d)(i) LDPB=エーま-0・ = STC の・・・/	
Su = 112=(3)12	50.50 M 54.50 CP 52.22.25 B 65.54 CP 1.55 CP 45.50 CP 45.50 CP 45.50 CP 1.55 CP 45.50 CP	
= -(77026	$\therefore A = \frac{1}{2} \times bol Sin(\underline{S}^{T} - \Theta).$	
: T <sub>12</sub> = -5 3 12 97-1-17702L) = -354271	ii) dA ½ loc( los(당근-0)	
13 13 14 W 1 6 52 C	Statet when do =0	
$\frac{c)}{chL} \frac{d}{(\cos 2t)^{-1}} = -(\cos 3t)^{2} v - 51t$ $= \frac{51tr7L}{\cos^{2} 2L}$	$\frac{\chi}{\cos(\frac{\sqrt{10}}{2}-0)} = 0$ $\cos(\frac{\sqrt{10}}{2}-0) = 0$	######################################
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	when $\theta = \frac{\pi}{3}$			The state of the s	
	$\frac{d^2A}{d\theta^2} = \frac{1}{2}bdS$	in (I)			TINN OLOMOOOD GOODS
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# Question 15 (wont) 01)11) A, = (6460 x (-005 - 101 x 1.005 7 OR A, = (1460-101) x (-005 -387 A, = 16460×1:0052 - 101×1:0052 - 1:01 × 1:005 - 387 VI.005 - 387 An = 16460 × 1.005" - 101×1:005" - 101×1:005" - --- -101×1:005 -387 × 1.005 "-----387 $0 = 16460 \times (.005^{h} - 101(1.005^{n} + 1.005^{n-1} + ... + 1.005)$ $-387(1.005^{n} + 1.005^{n-2} + ... + 1)$ 0 = 16460 × 1:005 - 101 (1:005 (1:005 12.1) 0=16460 x1-005" - 20301 (1-005"-1) - 77400 (1:005°1-1) 0 = 16460 × 1:0057 - 97701 (1:0057-1) 0=16460x (0057 = 97761x10054 + 97761 $-97701 = -8(241 \times (.005^{n})$ 97701 . 1.005h. $n = \log_{1005} \left( \frac{97701}{81241} \right)$ $n = 36.99 \dots$ 1 37 payments